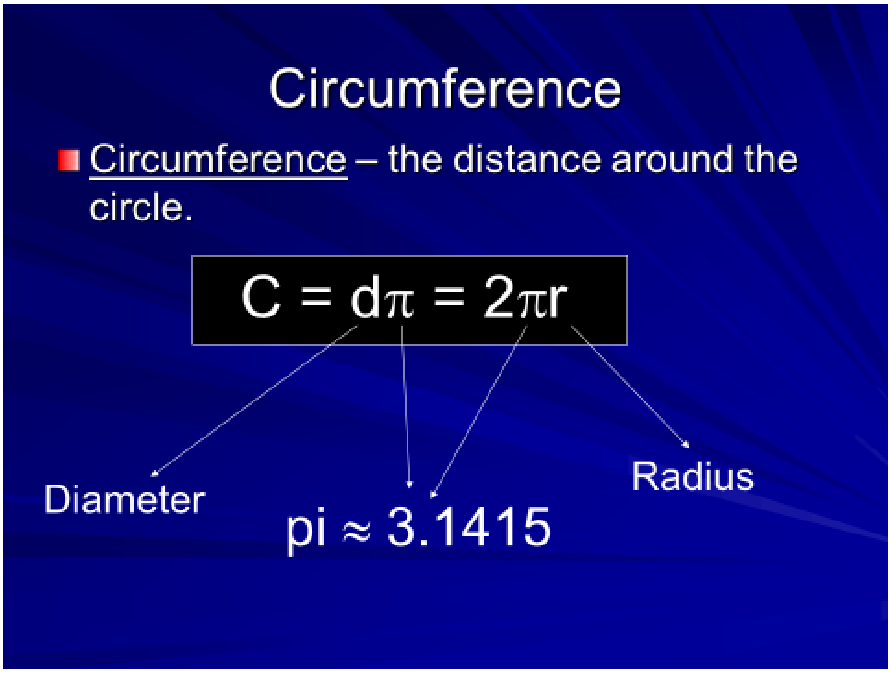
Section 10-7
Area of circles, sectors and segments

Lesson objectives: to learn how to compute the area of circles, sectors and segments and how to apply those properties to solve problems.



$$\text{Length of } \overparen{AB} = \frac{m\overparen{AB}}{360} \cdot 2\pi r$$

Arc measure (in degrees)

Circumference

Ex: An arc of 40° represents 40/360 or 1/9 of the circle.

We call this the “circle fraction” as it represents the portion of the circle in question.

Area of a circle

$$A = \pi r^2$$

Remember: area is in square units!

Example: area of a circular mat

Since the diameter of the region is 32 ft, the radius

$$A = \pi r^2 \quad \text{Use the area formula.}$$
$$= \pi(16)^2 \quad \text{Substitute 16 for } r.$$
$$= 256\pi \quad \text{Simplify.}$$
$$\approx 804.2477193 \quad \text{Use a calculator.}$$

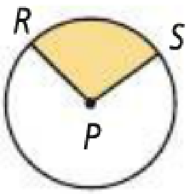
What are the units?

ft^2.

Sector of a circle: what is it?

A **sector of a circle** is a region bounded by an arc of the circle and the two radii to the arc's endpoints. You name a sector using one arc endpoint, the center of the circle, and the other arc endpoint.

The area of a sector is a fractional part of the area of a circle. The area of a sector formed by a 60° arc is $\frac{60}{360}$, or $\frac{1}{6}$, of the area of the circle.

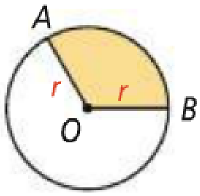


Sector *RPS*

Theorem 10-12: Area of a sector

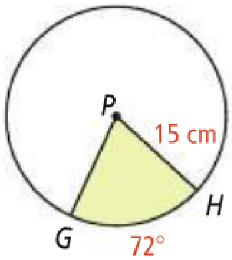
The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

$$\text{Area of sector } AOB = \frac{m\overparen{AB}}{360} \cdot \pi r^2$$



Example: area of a sector

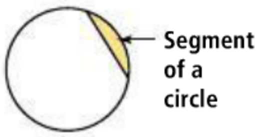
$$\begin{aligned}\text{area of sector } GPH &= \frac{m\overparen{GH}}{360} \cdot \pi r^2 \\ &= \frac{72}{360} \cdot \pi(15)^2 \\ &= 45\pi\end{aligned}$$



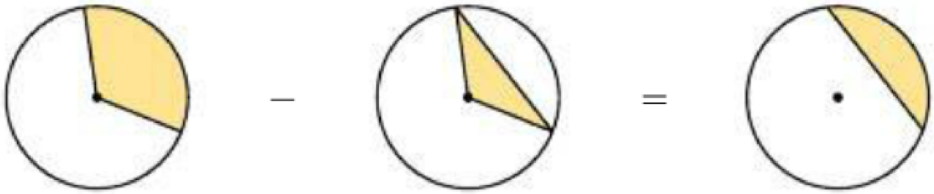
The area of a sector formed by a 72° arc is $\frac{72}{360}$, or $\frac{1}{5}$, of the area of the circle.

Area of a segment: what is a segment?

A part of a circle bounded by an arc and the segment joining its endpoints is a **segment of a circle.**



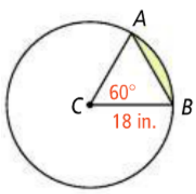
To find the area of a segment for a minor arc, draw radii to form a sector. The area of the segment equals the area of the sector minus the area of the triangle formed.



Area of sector − Area of triangle = Area of segment

Example: area of a segment

Step 1: find the area of the sector

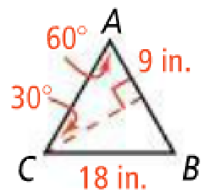


$$\text{area of sector } ACB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$$

$$= \frac{60}{360} \cdot \pi(18)^2$$

$$= 54\pi$$

Step 2: find the area of the triangle



$$\text{area of } \triangle ACB = \frac{1}{2}bh$$

$$= \frac{1}{2}(18)(9\sqrt{3})$$

$$= 81\sqrt{3}$$

Step 3: subtract the area of the triangle from the area of the sector

$= 54\pi - 81\sqrt{3}$ Substitute.

≈ 29.34988788 Use a calculator.

The area of the shaded segment is about 29.3 in.^{2}

How are the arc length and sector area equations similar?

$$\text{Length of } \widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$$

$$\text{Area of sector } AOB = \frac{m\widehat{AB}}{360} \cdot \pi r^2$$

Each of the regular polygons in the table has radius 1. Use a calculator to complete the table for the perimeter and area of each polygon. Write out the first five decimal places.

Polygon	Number of Sides, n	Length of Side, s	Apothem, a	Perimeter ($P = ns$)	Area ($A = \frac{1}{2}ap$)
Decagon	10	2(sin 18°)	cos 18°	6.18033 . . .	2.93892 . . .
20-gon	20	2(sin 9°)	cos 9°	■	■
50-gon	50	2(sin 3.6°)	cos 3.6°	■	■
100-gon	100	2(sin 1.8°)	cos 1.8°	■	■
1000-gon	1000	2(sin 0.18°)	cos 0.18°	■	■

Look at the results in your table. Notice the perimeter and area of an n-gon as n gets very large. Now consider a circle with radius 1. What are the circumference and area of the circle? Explain your reasoning.